

**Federal state educational budgetary institution  
of higher education  
"FINANCIAL UNIVERSITY  
UNDER THE GOVERNMENT OF THE RUSSIAN FEDERATION"  
(Financial University)**

**Department of Data analysis,  
Decision-making and Financial technologies**

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**Financial Mathematics**

**SYLLABUS**

*Level of Study: Bachelor's Degree*

*Field of Study: 38.03.01 - "Economics"*

*Study Program: International Finance (in English)*

Moscow -2020

## Syllabus

### 1. Name of a subject: Financial Mathematics

### 2. Mapping of learning outcomes (list of competences), with the relevant indicators described and subject learning outcomes indicated

Table 1

Competence code	Competence	Competence development indicators <sup>1</sup>	Learning outcomes (skills <sup>2</sup> , and knowledge) and indicators that show competence development
<b>ПКХ-3</b>	Ability to apply mathematical methods to solve standard professional financial and economic problems, interpret the obtained mathematical results		<i>Knowledge:</i> mathematical methods for the solution of the standard professional economic and financial problems. <i>Skills:</i> can use knowledge in the field of information technology to build mathematical models for solving applied problems.
<b>ПКХ-6</b>	Ability to offer solutions to professional problems in changing financial and economic conditions		<i>Knowledge:</i> quantitative methods of analysis of initial data necessary for solving professional tasks in changing financial and economic conditions. <i>Skills:</i> can apply computational methods of data analysis necessary for solving professional tasks in changing financial and economic conditions

### 3. Place of the subject in the curriculum

The discipline "Financial mathematics" is included in the module of mathematics and computer science (information module) of the mandatory disciplines of the educational standard of the Financial University in the Field of Study of bachelor training 38.03.01-"Economics", and belongs to the class of disciplines focused on the formation of General cultural and professional competencies of graduates in the field of data processing in Economics and Finance.

In the course of studying the discipline, you get acquainted with the basic concepts, methodologies, models, methods, techniques and technologies of modern financial mathematics, knowledge of which will be used as the basis for the formation of

<sup>1</sup> To be filled in when the updated Financial University educational standards and federal state educational standards of higher education "3++" are implemented.

<sup>2</sup> Skills are described when the Financial University educational standards of the 1<sup>st</sup> generation and federal state educational standards of higher education "3+" are implemented.

professional competencies of a graduate of the Field of Study "Economics". At the same time, students gain experience in practical use of the studied technologies in practical tasks related to financial technologies.

The discipline "Financial mathematics" lays the Foundation for assessing financial flows in conditions of uncertainty, for making decisions on the formation of a portfolio of financial instruments and managing such a portfolio, provides initial information about the risk of financial instruments and forecasting financial series.

#### **4. Workload in credits and academic hours, with class work (lectures and seminars) and self-study indicated**

The data are presented in the form of a table.

Table 2

<b>Type of work</b>	<b>Total (in credits and hours)</b>	<b>5 Semester (in hours)</b>
<b>Overall workload</b>	<b>4 credits 144</b>	<b>144</b>
<i>Class work</i>		
<i>Lectures</i>	<b>16</b>	<b>16</b>
<i>Seminars, practicals</i>	<b>54</b>	<b>54</b>
<i>Self study</i>	<b>94</b>	<b>94</b>
Formative assessment	<b>Control work</b>	<b>Control work</b>
Summative assessment	<b>Exam</b>	<b>Exam</b>

#### **5. Subject content (with the thematic components indicated).** This subsection

##### **Section 1. Fundamentals of classical financial mathematics.**

Topic 1.1. Day count convention. ACT and Banker`s rule. Financial system ACT/ACT, ACT/365, ACT/360, ACT/360.

Topic 1.2. Simple and compound interest: types of interest rates, effective interest rate, discount rate and bill accounting, inflation, real interest rate.

Recommended sources: [1], [2], [4], [5].

##### **Section 2. Cash flow streams**

Topic 2.1. Cash flow streams. Value of a cash flow stream. Present value and future value of the stream. Duration of the Cash flow stream and its properties. Recommended sources: [1], [2], [4], [5].

Topic 2.2. Investment projects: numerical indicators of the effectiveness of investment projects. Net present value. Comparison of investment cash flow streams. Internal rate of return. Recommended sources: [1], [2], [4], [5].

##### **Section 3. Annuities**

Topic 3.1. Ordinary Annuities and Annuities Due. Characteristics of Annuities and their finding. Perpetuity. P- Annuities. Consolidation and replacement of Annuities.

Topic 3.2. Loan calculations. Regular loan payment, minimum loan term, a (partial) early repayment, loans with a grace period. Recommended sources: [1], [2], [4], [5], [8]

#### **Section 4. Bonds**

Topic 4.1. Bonds: mathematical model of bonds. The main characteristics of the bond. Yield to maturity and its approximation. Accrued Interest. Dirty and clean bond price.

Recommended sources: [1], [2], [7], [8].

Topic 4.2. Hedging interest rate risk: bond duration and its properties, bond portfolio duration, immunization theorem, bond portfolio management. Recommended sources: [1], [2], [7].

#### **Section 5. Portfolio Analysis**

Topic 5.1. Optimal portfolio of risky assets: return and risk, a portfolio of two securities, a set of acceptable portfolios, building an optimal portfolio. Minimal risk portfolio. The portfolio of zero risk. A minimal risk portfolio with a specified return. The maximum expected return portfolio.

Recommended sources: p. 8, [1], [2], [3], [6].

Theme 5.2. Portfolio of more than two securities. Minimal risk portfolio. Markovitz model, building an optimal portfolio in the presence of a risk-free security. Finding the tangent portfolio. Utility function. Decision-making depending on the attitude to risk.

Recommended sources: [1], [2], [3], [6]

### **6. List of teaching and methodological materials needed for the students self-study**

#### **6.1. List of questions for student self-study and types of out-of-class activities**

The section lists types of out-of-class activities that correspond to items in the subject content description.

There is a list of questions the students should answer while working independently.

<b>Itemized subject content</b>	<b>Questions the students should answer within the self-study process</b>	<b>Types of out-of-class activities</b>
1. Fundamentals of classical financial mathematics	The scheme of repayment of the loan. Inflation and its impact on the performance of investment projects Sources: Internet resources.	Working with educational literature. The solution of standard tasks. Analysis of questions on the topic of the lesson. Completing homework for each lesson.
. 2. Bonds	Methods for determining the time structure of interest rates. Sources: Internet resources.	Working with educational literature. The solution of standard tasks. Analysis of questions on the topic of the lesson. Completing homework for each lesson.
3. Portfolio Analysis	Methods for building optimal portfolios. Sources: Internet resources.	Working with educational literature. The solution of standard tasks. Analysis of questions on the topic of the lesson. Working with sources and searching for information on the Internet. Completing homework.
4. Derivative financial instrument	Continuous random processes Ito Integral Stochastic differential equation Numerical methods for calculating the price of derivatives Sources: [1], [3], Internet resources.	Working with educational literature. The solution of standard tasks. Analysis of questions on the topic of the lesson. Completing homework for each lesson.

## **6.2. List of questions/assignments/topics for students' preparation to formative assessment**

Formative assessment is carried out during the educational process and control of independent work of students, based on the results of the control work.

### **The main forms of formative assessment are:**

1. discussion of issues and tasks included in the practical training plans;
2. solving problems and discussing them;
3. execution of control tasks and discussion of results;
4. control work.

### **Subject of the control work tasks**

\* Percentage calculations.

- \* Calculating the numerical characteristics of cash flows.
- \* Rental calculations.
- \* Numerical characteristics of bonds.
- \* Hedging interest rate risk.
- \* Portfolio analysis.

### Example of control work

#### 1

- 1) Find the amount paid on the bill discounted under the scheme of compound interest 22.01.15, if Face is 1000 rubles and maturity date is 11.02.2018, discount rate  $d$  is 12 %;
- 2) Find an effective interest rate if the amount of 60000 rubles was placed for six months at a simple interest rate of 12%, and then at a compound interest for 1 year at the rate of 9% compounded 4 times per year;
- 3) Find a) PV, FV,  $V(1,5)$ ;  $V(2,5)$ ; b) duration of the Cash Flow Stream  
(2000; 1); (1200; 2); (1500; 2,5); (2500; 3);  $i = 8\%$ .
- 4) Compare the Cash Flows  
(- 1500; 0); ( 1000; 1); (1500; 2,5); (2000; 3);  
(- 1500; 0); ( 1200; 2); (1800; 3,5); (2400; 4);  
if a)  $i = 8\%$ ; b)  $i = 20\%$ .
- 5) Consolidate a 5-year annuity with an annual payment  $R_1 = 2500$  and a 2-year annuity with an annual payment  $R_2 = 4000$  using a 4-year annuity if the annual interest rate  $i = 12\%$  (Find  $R_3$ ).

#### 2

- 1) Find the clean price of a 6-year bond if the face value  $F = 1000$ , 2,3 years have passed since the issue, the coupon paid 2 times per year and **I** the yield to maturity  $i = 12\%$ , the coupon rate  $c = 9\%$ ; **II** the yield to maturity  $i = 9\%$ , the coupon rate  $c = 12\%$ .
- 2) Find the duration of a 5-year bond with the face  $F = 1000$  if **I**  $P = 1017,35$ , the coupon rate  $c = 11\%$ , **II**  $P = 965,29$ , the coupon rate  $c = 8\%$ ; the coupon paid annually.
- 3) Find the portfolio of the maximum return of two securities, its expected return and risk if  
**I** the expected returns  $\mu_1 = 25$ ,  $\mu_2 = 12$ , the risks  $\sigma_1 = 10$ ,  $\sigma_2 = 4$ ;  
**II** the expected returns  $\mu_1 = 22$ ,  $\mu_2 = 8$ , the risks  $\sigma_1 = 7$ ,  $\sigma_2 = 3$ ;  
the correlation coefficient  $\rho = -0,5$  and the risk of the portfolio at most 5.
- 4) Find the minimum risk portfolio, its expected return and risk if the return vector and the covariance matrix are respectively equal

$$\text{I } \mu = \begin{pmatrix} 5 \\ 3 \\ 8 \end{pmatrix}; V = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 12 \end{pmatrix}; \text{II } \mu = \begin{pmatrix} 3 \\ 6 \\ 12 \end{pmatrix} V = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}.$$

5) Find the minimum risk portfolio of two securities, its expected return and risk if  
**I** the expected returns  $\mu_1 = 18$ ,  $\mu_2 = 11$ , the risks  $\sigma_1 = 6$ ,  $\sigma_2 = 3$ ;  
**II** the expected returns  $\mu_1 = 25$ ,  $\mu_2 = 10$ , the risks  $\sigma_1 = 8$ ,  $\sigma_2 = 5$ ;  
the correlation coefficient  $\rho = 0,5$  and the return of the portfolio at least 12.

### 7. Mandatory and optional reading list

1. Соловьев В. И. Финансовая математика: учебное пособие / В. И. Соловьев. — Москва: КНОРУС, 2016. — 176 с. — То же [Электронный ресурс] — Режим доступа: <https://www.book.ru/book/919835>

2. Бабайцев В.А. Математические методы финансового анализа: учебное пособие / В.А. Бабайцев, В.Б. Гисин. — Москва: Финансовый университет, 2011. — 200 с.

3. Гисин В.Б. Математические основы финансовой экономики [Электронный ресурс]: учебное пособие / В. Б. Гисин, А. С. Диденко, Б. А. Путко. — Москва: Финансовый университет. — 2017. — 180 с. — Режим доступа:

[https://portal.fa.ru/Files/Data/3b25572d-6968-44f8-95c0-1423ae1981af/Uchpos\\_Mathmethodfinan\\_17.pdf](https://portal.fa.ru/Files/Data/3b25572d-6968-44f8-95c0-1423ae1981af/Uchpos_Mathmethodfinan_17.pdf)

### Optional reading list

4. Аль-Натор М.С. Основы финансовых вычислений (факты, формулы, примеры, задачи и тесты). Ч. 1: учебное пособие / М. С. Аль-Натор, Ю. Ф. Касимов, А. Н. Колесников. — Москва: Финуниверситет, 2012. — 159 с.

5. Аль-Натор М.С. Основы финансовых вычислений (факты, формулы, примеры, задачи и тесты). Ч. 2: учебное пособие / М. С. Аль-Натор, Ю. Ф. Касимов, А. Н. Колесников. — Москва: Финуниверситет, 2013. — 176 с.

6. Аль-Натор М.С. Основы финансовых вычислений (факты, формулы, примеры, задачи и тесты). Ч. 3: учебное пособие / М. С. Аль-Натор, Ю. Ф. Касимов, А. Н. Колесников. — Москва: Финуниверситет, 2014. — 149 с.

7. Аль-Натор М.С. Основы финансовых вычислений (факты, формулы, примеры, задачи и тесты). Ч. 4: учебное пособие / М. С. Аль-Натор, Ю. Ф. Касимов, А. Н. Колесников .— Москва: Финуниверситет, 2015 .— 167 с.

8. Luenberrger D.G Investment Science / Oxford University Ptes, Oxford, 1998 .— 454 p

**8. List of IT resources, incl. the list of software, information and reference systems (as appropriate).**

**8. 1. Software:**

1. Windows, Microsoft Office software (Exel);

**8.2. Databases and information and reference systems**

E.g.

1. Information and educational portal of the Financial University under the Government of the Russian Federation <http://portal.ufrf.ru/>

2. Website of the Department of data analysis, decision-making and financial technologies.

3. Electronic library of the Financial University (EB) <http://elib.fa.ru/> (<http://library.fa.ru/files/elibfa.pdf>)

4. Shiryayev A. N. Fundamentals of stochastic financial mathematics. Volume 1. Facts. Models. - Moscow: MCMNO, 2016. - 440 p. — <http://t-library.ru/showBook.php?id=2294>

5. Electronic library system BOOK.RU <http://www.book.ru>

6. Electronic library system of publishing house "LAN" <https://e.lanbook.com/>

7. Electronic library system "University library ONLINE" <http://biblioclub.ru/>

8. Electronic library system Znanium <http://www.znaniy.com>